**Pythagorean Theorem**

**Section 1**

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| 00:00:00 | TEACHER: Take a look at this chessboard. |
| 00:00:02 | It has a total of 64 squares. And running along each of these sides is 8 squares or a distance of 8. Since it has 64 squares, and one of its sides is 8, 8 squared must be 64. So 64 is called a perfect square. We need to understand this concept in order to answer the question, what are the properties of right triangles? |

**Section 2**

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| 00:00:00 | TEACHER: Let's take a look at perfect squares. |
| 00:00:02 | A perfect square is a number that is the result of squaring a natural or whole number. Take a look at the blue square on the left. It's made up of 16 squares. And 16 is a perfect square because it is the result of multiplying 4 times 4 together. Notice, the length of this side is 4, and the length of this side is 4. |
| 00:00:23 | And 4 times 4, which is 4 squared, is equal to 16. That's why 16 is a perfect square. What about the orange square to the right? It's made up of 36 squares total. And it is also a perfect square because it is the result of multiplying 6 with 6 or taking 6 squared. You see 6 times 6 is equal to 6 squared. And 6 squared is equal to 36. |
| 00:00:49 | And that's why 36 is a perfect square. So let's take a look at perfect squares verses not perfect squares. The perfect squares first. You see, 4 is the result of 2 squared. 25 is the result of 5 squared. And 64 is the result of 8 squared. Now what about the not perfect squares? |
| 00:01:11 | Like, 18? Well, 4 squared is 16, isn't it? And that's less than 18. But the very next integer past 4 is 5. And 5 squared is 25, which is bigger than 18. 18 is between 2 perfect squares. So it can't be a perfect square itself. What about 42? |
| 00:01:31 | Well, 42 is between 6 squared, and 7 squared. So therefore, 42 is not a perfect square. Now finally, what about 80? 80 is between 8 squared and 9 squared, which is 81. Since 80 is between 2 perfect squares, it can't be a perfect square itself. |

**Section 4**

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| 00:00:01 | TEACHER: I remember flying on a plane once and looking down |
| 00:00:03 | outside the window. And when you're that high up, you see how a lot of the roads tend to make these geometric shapes. In this case, in the case of the image, we see we have this shape of a triangle here. Now, I wonder if there's a relationship between the sides of the triangle? Let's keep answering the question, what are the |
| 00:00:23 | properties of triangles? And see if we can find out. |

**Section 5**

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| 00:00:00 | TEACHER: Let's take a moment to talk about perfect squares |
| 00:00:02 | of right triangles. So squares can be created from the side lengths of a right triangle. In this case, the blue square was created by squaring the side length of 3 of this right triangle. So this side is 3. And this side is length 3. So the area of the square is 3 squared, or 9. |
| 00:00:19 | Looks like a 7, doesn't it? But it's actually a 9. And the orange square was created by squaring the side length of 4 from the right triangle. So this side is length 4. This side is length 4. And the area of the square is 4 squared, which is equal to 16. |
| 00:00:36 | Notice that 9 plus 16 is equal to 25. Now I wonder, is there a relationship between these two areas, the area of the blue square and the orange square and the area of the green square? Let's investigate that. So we know that as far as the blue square is concerned, the area is 9. |
| 00:01:01 | And the orange square, we know, has an area of 16. And we've already said that 9 plus 16 is equal to 25. Well, if I take a look at the green square, I'll notice that it was created by the side length of 5 from the right triangle. So this side length is 5 and this side length is 5, which gives me the area for the green square of 25. So what's happening here is that the sums of the two |
| 00:01:35 | squares that were created from the legs of the right triangle is actually equal to the square of the square created by the hypotenuse of the right triangle. This result relating the sides of a right triangle is known as the Pythagorean theorem. It states that for any right triangle with the legs a and b and hypotenuse c, this relationship holds, a squared plus b squared will have to be equal to c squared. |
| 00:02:04 | And we can verify this relationship using the values that we've been given for the side length of this right triangle. So 3 substituted in for a, 4 substituted in for b, and 5 substituted for c, well, 3 squared is 9. 4 squared is 16. And this is equal to 5 squared equals 25. |
| 00:02:32 | 9 plus 16 is 25, and 25 equals 25. Notice that I could have rearranged my substitution for a and b. I would have just ended up with 16 plus 9 instead of 9 plus 16. Either way, we can see that the side lengths of this right triangle satisfy the Pythagorean theorem. |

**Section 8**

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| 00:00:00 | TEACHER: Let's take a look at the sides of a particular |
| 00:00:02 | right triangle. So let's examine a right triangle ABC with a side length a equals 8 that is opposite angle A, a side length b, which is equal to 15 and opposite angle B, and the side length c, which is equal to 17, opposite angle C. Now take a look at these squares that are formed by the sides of this right triangle. Something interesting happens if we just continue to make |
| 00:00:26 | each of these squares bigger and bigger and bigger. What happens is that the squares that are created by squaring the lengths of the legs of the right triangle, their sum will always equal the square that was created by squaring the length of the hypotenuse of the right triangle. This result is the Pythagorean theorem. The diagram shows how 8, 15, and 17 satisfy the Pythagorean theorem, a squared plus b |
| 00:00:55 | squared is equal to c squared. When three whole numbers form the sides of a right triangle, they're called Pythagorean triples. So in this case, 8, 15, and 17 represent a Pythagorean triple. Knowing Pythagorean triples can help us determine when a right triangle is a right triangle and also unknown side lengths of a right triangle. |
| 00:01:17 | Let's take a closer look at Pythagorean triples. A Pythagorean triple is a set of 3 positive integers that satisfy the Pythagorean theorem and are possible side lengths of a right triangle. Side lengths 3, 4, and 5 is a Pythagorean triple because 3 squared plus 4 squared is equal to 5 squared. Let's see if we can verify 5, 12, and 13 as a Pythagorean triple. |
| 00:01:39 | So we need to substitute these values in for a, b, and c into the Pythagorean theorem. So 5 will be substituted for a. 12 will be substituted for b. And 13 will be substitute in for c. 5 squared is 25. 12 squared is 144. And 13 squared is 169. |
| 00:02:01 | 25 plus 144 is 169, and that is equal to 169. So 5, 12, and 13 have satisfied the Pythagorean theorem and therefore is a Pythagorean triple. What about 7, 24, and 25? So 7, substitute in for A. 24, substitute in for b. And 25, substitute in for c. 7 squared is 49. 24 squared is 576. |
| 00:02:30 | And 25 squared is 625. 49 plus 576 is 625, which is equal to 625, of course. Therefore 7, 24, and 25 is a Pythagorean triple. Now, we can also use whole number multiples of known Pythagorean triples to develop other Pythagorean triples. So let's say you're taking a look at the triple 3, 4, 5. If I multiply this by, let's say, 3, well this would become the a 9. |
| 00:03:01 | 4 would become a 12. And 5 would become 15. The question is, did I just form a new triple by doing that? Well, let's verify. Is 9 squared plus 12 squared equal to 15 squared? That's really the question I'm asking. Well, 9 squared is 81. 12 squared is 144. |
| 00:03:21 | And 15 squared is 225. 81 plus 144 is 225, which is obviously equal to 225, so yes. 9, 12, 15 is a triple. And we found that triple by multiplying the known triple 3, 4, 5 by a whole number. Since I can choose any whole number to multiply by, 3, 4, 5, 6, in creating these triples, I know that there are |
| 00:03:49 | infinitely many Pythagorean triples. |

**Section 10**

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| 00:00:00 | TEACHER: Take a look at this image here. |
| 00:00:02 | Do you see any right triangles? How about the masts and the lines that connect to these sailboats? It turns out right triangles are everywhere in the world around us. And now, you know the Pythagorean theorem, which is super cool. So we're going to try to apply these ideas and solve some |
| 00:00:22 | real world problems as we answer the question, what are the properties of right triangles? |

**Section 11**

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| 00:00:00 | TEACHER: Let's take a look at a real-world use of the |
| 00:00:02 | Pythagorean Theorem. So Stella purchases a small plot of land with dimensions of 30 yards, 16 yards, and 34 yards. The question is, how can we determine if this triangular lot forms a right triangle? Well, if it does form a right triangle, I know that the side lengths 16, 30, and 34 must satisfy the Pythagorean Theorem a squared plus b squared is equal to c squared. |
| 00:00:27 | So I'll substitute 30 in for a, 16 in for b, and 34 in for c. So what is 30 squared? 900. 16 squared is 256. And 34 squared is 1,156. Well, 900 plus 256 is 1,156, which is equal to 1,156. This is a true statement. |
| 00:00:59 | And that tells me that these side lengths satisfy the Pythagorean Theorem. Therefore, her plot of land is in the shape of a right triangle. |

**Section 13**

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| 00:00:00 | TEACHER: Let's take a look at a real-world problem in using |
| 00:00:03 | Pythagorean Triples. So Carson found a piece of wood he wants to use to make a table top. The wood measures 40 inches by 10 inches. And the diagonal of the wood is 41 inches. The question we want to answer is, is this piece of wood rectangular? If it is rectangular, I know the interior angle measures |
| 00:00:20 | are all 90 degrees, which would give me 2 right triangles created by the diagonal of 41 inches. So if they're all 90 degrees, I know this would be a right triangle. And if it is a right triangle, 10, 40, and 41 must satisfy the Pythagorean Theorem a squared plus b squared is equal to c squared. If these values do not satisfy the Pythagorean Theorem, then |
| 00:00:49 | this piece of wood he has is simply a non-rectangular parallelogram. So let's check. So 10 will be substituted in for a. 40 will be substituted in for b. And 41 will be substituted in for c. And we want to know is 10 squared plus 40 squared equal to 41 squared? |
| 00:01:07 | Well, 10 squared is 100. 40 squared is 1,600. Is this sum equal to 41 squared, which is 1,681? Well, clearly I see 1,600 plus 100 is 1,700. And this is not equal to 1,681. So the piece of wood he has is a non-rectangular parallelogram. Now if we knew Pythagorean Triples, that would help us |
| 00:01:37 | really quickly decide whether or not this piece of wood was rectangular. The numbers we were dealing with are not a Pythagorean Triple. But they are close to the Pythagorean |