**Lesson Question**

# Lesson Goals

## Use

of change to determine initial value.

**Examine** different representations of linear functions to find rate of change and initial value.

## Explore

**Describe** how rate of change and initial value represent a situation.

**?**

**W2K**

.

**Words to Know**

*Fill in this table as you work through the lesson. You may also use the glossary to help you.*

|  |  |
| --- | --- |
|  | to be an example or expression of something |
|  | the value of the dependent variable produced when an equation processes an input |
|  | the output of a function when the input is 0 |
|  | in a function, the ratio of the change in the dependent value with respect to the change in the independent value |
|  | a value that is transformed by a process and becomes output |

# Linear vs. Non-Linear Functions

## Functions

Graph is a line

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | 8 | *y* |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 6 |  | 3 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | 2 |  |  | 4 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | *x* |
|  | 8 |  | 4 |  |  |  | 4 |  | 8 |
|  |  |  |  | 4 |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |
| 2 |  |  | 8 |  |  |  |  |  |
|  |  |
|  |  |  |  |  |  |  |  |  |  |

Rate of change is .

## Functions

Graph is *not* a line

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | 8 | *y* |  |  |  |  |  |
|  |  |  |  |  |  |  | 12 |  |  |
|  |
|  |  |  |  |  |  |  |  |  |
| 2 |
|  |  |  |  | 4 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | *x* |
|  | 8 |  | 4 |  |  |  | 4 |  | 8 |
|  |  |  |  | 4 |  |  |  4 |  2 |  |
|  |  |  |  |  |  |  | 2 |  |  |
|  |  |  |  | 8 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

Rate of change is non-constant.

**Slide**

gal

hr

Rate of change =

gallons

Initial value =

Aiko is emptying her 1,600 gallon pool at a rate of 210 gallons an hour. What is the initial value?

.

**Initial Value**

**Initial value** is the starting value of a function when the **input** is

**Initial Value in a Table**

**EXAMPLE**

Namazzi started her savings plan a few months ago and created a table to monitor her progress.

What does the initial value **represent**?

Input

Output

deposit

What is the input of the initial value?

What is the **output** of the initial value?

$

**2**

|  |  |
| --- | --- |
| **Number of Months** | **Amount Saved ($)** |
| 0 | 55 |
| 1 | 67 |
| 2 | 79 |
| 3 | 91 |
| 4 | 103 |

**Slide**

**How to Find an Initial Value Using a Table**

* Find the **rate of change**, by calculating the common difference between consecutive outputs.
* Subtract the common difference repeatedly until the corresponding

−12

−12

input is

.

* Use a multiple of the

difference.

−12

103 − 4(12) =

**Finding Initial Value Using a Table**

Gregor adds the same amount of money to his piggy bank each month. The table below shows the total amount of money in his bank at the end of months 3, 4, and 5. Find the amount of money in the bank when he started saving.

*Fill the missing values in the table.*

Rate of change =

, Initial value = 5

41 − 3(12) =



|  |  |
| --- | --- |
| **Number of Months** | **Amount Saved ($)** |
| 3 | 41 |
| 4 | 53 |
| 5 | 65 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Month** | 0 | 1 | 2 | 3 | 4 | 5 |
| **Saved ($)** |  |  |  | 41 | 53 | 65 |

**4**

|  |  |
| --- | --- |
| **Number of Months** | **Amount Saved ($)** |
| 0 | 55 |
| 1 | 67 |
| 2 | 79 |
| 3 | 91 |
| 4 | 103 |

**Slide**

# Find Rate of Change from a Graph

**EXAMPLE**

An arcade gives a certain number of free tokens to first-time customers. The graph shows the number of tokens a customer received for $3, $4, $5 and $6. Assuming the relationship is linear, find the number of tokens a customer would receive per dollar.

48 *y*

44 6



40 1

36

(6, 44)

(5, 38)

32 6 

6

28 1

**Tokens**

24

20

16

12

8

4

(4, 32)

(3, 26)

*x*

1 2 3 4 5 6 7 8

**7**


## Dollars Spent

Rate of change =

For each spent the customer would receive 6 tokens.

**Slide**

# How to Find an Initial Value Using a Graph

* Find the rate of change:
* Rise over run is needed to move between ordered pairs.

6 

1

* + Repeat rise over run, moving right to left, until *x* = 0.

48 *y* 44 40 36 32 28 24 20 16 12

**Tokens**

8

4

(6, 44)

(5, 38)

(4, 32)

(3, 26)

*x*

1 2 3 4 5 6 7 8

**9**


## Dollars Spent

**Slide**

# Use Rate of Change to Determine Initial Value

The graph to the right shows the linear relationship between dollars spent and the number of tokens received. Use the fact that a customer receives 6 tokens for each dollar paid to determine the initial number of free tokens given.

*Plot (2, 20), (1, 14), and (0, 8) on the graph.*

Initial value =

6 = 6

1

*x* = 0

*y* = ?

48 *y* 44 40 36 32 28 24 20 16 12

8

**Tokens**

4

(6, 44)

(5, 38)

(4, 32)

(3, 26)

*x*

1 2 3 4 5 6 7 8

**9**


## Dollars Spent

When you walk into the arcade, you haven’t paid any money. You’re handed 8 free tokens.

**Slide**

# Analyze Data Sets

*x* = 0 *y* = 13

10

|  |  |
| --- | --- |
| **Day, *x*** | **Canned Goods, *y*** |
| 2 | 23 |
| 4 | 2 33 |
|  |  |  |
| 6 |  | 43 |
| 8 | 2 53 |

Rate of change = 2 =

10

10 Initial value =

*y*

(4, 33)

30

(3, 28)

26

22 (2, 23)

**Canned Goods**

18 (1, 18)

14 (0, 13)

10

*x*  0

6 *y*  13

2 *x*

5

Rate of change = 1 =

Initial value = 13

1 2 3 4 5 6

**12**


## Time (days)

The representation on the table and the graph have the same rate of change and same initial value. These two data sets represent the same linear function.

**Slide**

# Comparing Linear Relationships

Compare the different representations of the data below. Do they show the same information?

Rate of change =

2.5

1 =

11

10

*y*

|  |  |
| --- | --- |
| **Number of Weeks** | **Weight (lb)** |
| 4 | 16 |
| 5 | 18.5 |
| 6 | 21 |

9

8

7

**Weight (lb)**

6 (0, 5.7)

5

4

*x*

3

*y* 

2

(1, 8.2)

(2, 10.7)

Initial value = 16 − 4(2.5) =

5.7

 0

1

1 2 3

*x*

## Number of Weeks

Rate of change =

2.5

1

= 2.5

**14**

Initial value =

The rates of change for the data depicted by the table and graph are the same, but the initial values are different. The two data sets do not represent the same linear relationship.

**Summary** Constructing Linear Functions

**?**

What can a set of points tell you about a linear relationship?

**Lesson Question**

**Answer**

*Use this space to write any questions or thoughts about this lesson.*