**Linear Functions**

**Section 1**

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| 00:00:00 | TEACHER: We want to determine how to use data points from a |
| 00:00:03 | linear function to find an unknown. And now remember, a linear function has a constant rate of change. Nonlinear functions, they do not. So what we will do is we will take a constant rate of change and use that rate of change to find the initial value. Our lesson question is, what can a set of points tell you about a linear relationship? |

**Section 2**

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| 00:00:01 | TEACHER: The initial value is the starting value of a |
| 00:00:04 | function when the input is zero. Aiko is emptying her 1,600 gallon pool at a rate of 210 gallons per hour. We're asked, what is the initial value? Well, the initial value happens just once. Now, think about this. You have a swimming pool, and the swimming pool has a volume of 1,600 gallons. |
| 00:00:24 | So the initial value is the 1,600 gallons. And because the pool is being emptied at a rate of 210 gallons per hour, we have to be a little careful. We do have a constant rate of change. And the constant rate of change would be negative 210 gallons for each passing hour. Let's look at the concept of initial value a little bit more closely with the second example. |
| 00:00:54 | Namazzi is starting her savings plan a few months ago, and she created a table to monitor her progress. So initially, she deposits $55. And over a course of four months, it seems like with each passing month, she added $12. $12 after the first month, after the second month, after the third month, after the fourth month. This is an example of a constant rate of change. |
| 00:01:19 | For each passing month, we had an increase of $12 into the account. So we have a linear function. Well, our first question is, what does the initial value represent? Well, she initially deposited $55, so our representation would be that of the first deposit. What is the input of the initial value? |
| 00:01:45 | Well, the input is in terms of months, and the input of the initial value would be 0. What is the output of the initial value? Well, the output would be in terms of the money in the bank. Initially, she deposited $55. |

**Section 4**

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| 00:00:00 | TEACHER: Our goal is to find the initial |
| 00:00:02 | value using a table. The first step is to find the rate of change by calculating the common difference between consecutive outputs. Look at the table. If I look at consecutive outputs and subtract 55 from 67, I get 12. This is the same rate of change between any two outputs on the table that are consecutive. |
| 00:00:23 | Once I have this rate of change, which is 12, I will subtract the common difference repeatedly until the input is 0. What this means is that if I start with $103, subtract away $12 for each passing month, I can end up after four months finding that initial value to be $55. So I could speed the process up by using multiples of the common difference. |
| 00:00:50 | I can start with $103 and subtract away four deposits of $12 to find my initial value to be $55. Let us use these techniques to examine an application. Gregor adds the same amount of money to his piggy bank every month. The table on the left shows the activity in the piggy bank after months 3, 4, 5. What we want to do is find the initial amount that was in |
| 00:01:19 | that piggy bank. So first thing we do is we find the constant rate of change. Subtract any two consecutive outputs, and you get 12. So I can go ahead and say my rate of change is 12, so that after my third month, where I have $41 in the piggy bank, if I subtract $12, I would have $29 after the second month. Let's backtrack. |
| 00:01:45 | Subtract another $12. I would have $17 after the first month. Let's backtrack one more time and subtract $12, and we find that initially, when the input is 0, the output is 5. We have $5 in the piggy bank. What is our initial value? $5. We can speed the process up by saying that we have $41, and |
| 00:02:12 | we subtracted 3 amounts of $12 to find our initial value to be $5. |

**Section 6**

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| 00:00:00 | TEACHER: We're trying to determine how to use data |
| 00:00:03 | points from a linear function to find some unknown information. Up til now, we've used a constant rate of change to find an initial value. Now, can we turn to a graph and find a constant rate of change, and use it again to locate the initial value? Our lesson question is, what can a set of points tell you about a linear relationship? |

**Section 7**

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| 00:00:00 | TEACHER: Have you ever been to an arcade? |
| 00:00:02 | All those lights and noises going on around you? Well, some arcades work just like this one. This arcade gives a certain number of free tokens to first time customers. Now, after that, the person has to pay. For example, if the person paid $3, then they receive 26 tokens. If they pay $4, they would receive 32 tokens. |
| 00:00:24 | If they pay $5, they would receive 38 tokens. And if they pay $6, they would receive 44 tokens. We're asked to find the number of tokens a customer would receive per dollar. Well, we can do this by observing the graph. If I go from any discrete point to any other point on the graph, my rise would be 6 units, and my run would be 1. I would have a rise of 6 over a run of 1, and we'd find a |
| 00:00:54 | change of 6. Go from any other discrete point, we find the same rise over run to be 6 over 1. Well, this is an example of a linear function, because it has a constant rate of change. That constant rate of change is 6. And the interpretation is for each dollar spent, the customer would receive 6 tokens. |

**Section 9**

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| 00:00:00 | TEACHER: Let us learn how to find the initial value using |
| 00:00:03 | your graph. There are two steps. The first step is to find the rate of change. Now, we know how to do this. We take a point, and to get to the next consecutive point, in this case, we go down 6, left 1, down 6, left 1. So that rate of change is found with the calculation, negative 6 divided by negative 1, which is 6. |
| 00:00:27 | Our next step is to take our point and repeat the process of going down 6, left 1 until we actually find that initial value. So let's do this. Let's interpret the fact that we have 6 tokens for each dollar paid to mean we have 6 less tokens given to us for each less dollar spent. So what we want to do as our goal is to say to ourselves let |
| 00:00:55 | us find the initial value. And the initial value would occur when x is 0. So we go to our data point, and we go down 6 units, so down 6 from 26 is 20, left 1 unit, and now, we're at the point 2 comma 20. We'll continue the process, down 6 units from 20, left 1 unit, and we're at 1 comma 14. We're almost there, aren't we? |
| 00:01:28 | Let us go down another 6 units, left 1 unit, and we find ourselves at 0, 8. Our initial value is the output when x is 0. Our initial value's 8. Now, you know the interpretation, don't you? When you walk into the arcade, you haven't paid any money. You're handed 8 free tokens. |

**Section 11**

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| 00:00:00 | TEACHER: Up til now, we've been looking at data points on |
| 00:00:03 | graphs and tables. And we've been asking, is the relationship that we're seeing linear? If it's linear, what have we done? We have found the constant rate of change. And we've used the constant rate of change to determine the initial value. Well, now we'll be given two data sets. |
| 00:00:22 | And when we are, we're going to be looking for initial values. Our lesson question is, what can a set of points tell you about a linear relationship? |

**Section 12**

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| 00:00:01 | TEACHER: What you see in front of you is the representation |
| 00:00:03 | of two different data sets. Here's the question. Do the two data sets represent the same linear relationship? To answer this question we'll have to answer two questions. Does each data set have the same rate of change? Does each data set have the same initial value? Now let us turn to the table first. If I take a look at the table and look at consecutive |
| 00:00:25 | outputs, notice they differ by 10. The consecutive inputs differ by 2. So when I take the difference in the outputs of 10, and the difference of the inputs by 2, I find I have a constant rate of change of 5. Now turn to the graph, do we have a constant rate of change of 5 between two consecutive data points? Let's look at their outputs. |
| 00:00:52 | 18 minus 13 is 5. The difference in the inputs 1. Our rate of change is 5. This is true between any two consecutive data points, our rate of change is 5. We have the exact same constant rate of change for each representation. First question answered. |
| 00:01:15 | Now the second question is, do we have the exact same initial value for the two data sets? Well this is easier to answer on the graph. The initial value, well that's the output when x is 0. When x is 0 the output's 13. Let us see if 13 is the initial value for the table. Remember, whenever I increase by 2 units in the input, I increase the output by 10. |
| 00:01:43 | This means when I decrease 2 units in the input, I decrease the output by 10. It looks like when x is 0, y is 13 for the data represented on the table. So notice, we have the exact same initial value for the representation on the table and the graph. Same rate of change. Same initial value. |
| 00:02:07 | These two data sets represent the same linear function. |

**Section 14**

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| 00:00:00 | TEACHER: We have two sets of data. |
| 00:00:02 | Our question is, do the two sets of data represent the same linear relationship? Now suppose the sets of data represented the following. The input is the number of weeks old the puppy is. The output, well that's the weight of the puppy. So we can re-ask the question, is the puppy depicted by the table the same puppy as the puppy depicted by graph? How do we answer the question? |
| 00:00:26 | We ask two questions. Does the data shown in the table and the data shown in the graph have the same rate of change? And do they have the same initial value? Let's turn to the table first. Between any two consecutive outputs, the difference is 2.5. Well, because the difference between consecutive inputs is |
| 00:00:48 | 1, we can say the rate of change for the data depicted by the table is 2.5. Turn to the graph. Between any two points, our difference in outputs is 2.5. Between any two inputs, the difference is 1. It looks like our rate of change for the puppy depicted by the graph is 2.5. The data on the table, the data in the graph, we have the |
| 00:01:19 | exact same rate of change for that data. Same growth rate for the puppy. Second question. Second question is, do both data sets have the same initial value? Let's go to the graph first. When x is 0, what is the corresponding output? We can read that right off the graph, can't we? |
| 00:01:40 | It's 5.7. Initially, the puppy was 5.7 pounds, for the puppy depicted by the graph. Let's look at the puppy depicted by the table, and say to ourselves, what was the initial value? After a week four, the puppy was 16 pounds. We will subtract 4 times 2.5, because from birth four weeks passed. |
| 00:02:08 | Each week, the puppy gained 2.5 pounds. So we have 16 minus 10, which is 6. This puppy depicted by the table is 6 pounds at birth, not 5.7 pounds at birth. It's heavier. Two sets data do not represent the same linear relationship. |