**Analyzing Solutions**

**Section 1**

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| 00:00:00 | TEACHER: We're answering the question, how can you identify |
| 00:00:02 | the number of solutions of linear equations? You just reviewed using the distributive property and the properties of equality to solve multi-step linear equations with the variable on both sides. Now, let's explore more linear equations with different types of solutions. |

**Section 2**

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| 00:00:00 | TEACHER: Let's practice solving a |
| 00:00:01 | one-variable linear equation. So we'll use the properties of equality to solve this equation. 3 times the quantity, x plus 6 equals 2 times the quantity, x minus 3, plus 4x. So we'll start by using the distributive property to eliminate the parentheses. On the left, we'll distribute the 3, and on the right, we'll |
| 00:00:19 | distribute the 2 to the parentheses. This is going to give us the equation, 3 times x plus 3 times 6, which is 18, equals 2 times x minus 2 times 3, which is 6, plus 4x. The next step to solving this equation is going to be to combine like terms on either side of the equals sign. On the left, we have no like terms, so it stays 3x plus 18 equals. |
| 00:00:47 | On the right, we have 2x and 4x. Combining those gives us 6x. And we still have the minus 6. The next step to solving is to isolate the variable term on one side and the constants on the other side of the equal sign. So I'm going to start by using the subtraction property of equalities to subtract 3x from both sides. |
| 00:01:09 | This leaves me with the equation, 18 equals 6x minus 3x, which is 3x, minus 6. So since my variable is on the right hand side, I want to move my constants to the left. I'm going to do that by adding 6 to both sides. This gives me the equation, 24 equals 3x. So to finally isolate my variable, I need to divide out the coefficients. |
| 00:01:34 | This is the division property of equality. So I'm going to divide both sides by 3, which leaves me with 24 divided by 3 is 8. 3x divided by 3 is just x. So 8 equals x or x equals 8. And this is the solution to the equation. So to answer the question, what is the number of solutions to the one-variable equation, |
| 00:01:58 | there's only one solution. Let's quickly verify that our solution is correct by plugging it back into the equation and verifying that we get a true statement. So I'm going to replace all the xs in the original with 8. So I have 3 times 8 plus 6 equals 2 times 8 minus 3 plus 4 times 8. Using order of operations, I can go and simplify the left. |
| 00:02:24 | That's 3 times 14 equals 2 times 8 minus 3 is 5, plus 4 times 8 is 32. 3 times 14 is 42, 2 times 5 is 10, plus 32 gives me the true statement, 42 equals 42. So I know my solution, x equals 8, is correct. |

**Section 4**

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| 00:00:00 | TEACHER: Let's practice finding |
| 00:00:01 | the number of solutions. This equation has infinitely many solutions. So let's use the properties of equality to solve and see why. The first step to solving is distribute to remove the parentheses. So on the left, I'm going to distribute the 8. And on the right, I'm going to distribute the 4 into the parentheses. |
| 00:00:19 | This gives me 8 times x plus 32 equals 4 times x plus 32, which is 4 times 8, plus 4x. So step 2, we want to combine like terms on either side of the equal sign. The left has no like terms, so it's 8x plus 32. On the right side, we see we have 4x and 4x. Combining those gives us 8x. We still have the plus 32. |
| 00:00:49 | So step three, we want to isolate the variable term on one side and the constant on the other side. Well, let's start by using the subtraction property of equality to subtract 8x from both sides. Something interesting happens here. My variables cancel out on both sides and I'm left with 32 equals 32, a true statement. This means that any input value for x would solve this |
| 00:01:15 | equation, or satisfy the equation. So we have infinitely many solutions. So to answer the question, what is the number of solutions to this one-variable equation, infinitely many. Now, take a look at step two. 8x plus 32 equals 8x plus 32. We can see at step two that on the left, the coefficient to the variable and the constant are the same as on the |
| 00:01:43 | right-hand side of the equation. Whenever this happens, we have infinitely many solutions. |

**Section 6**

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| 00:00:00 | TEACHER: Let's practice finding |
| 00:00:01 | the number of solutions. You've seen examples of equations that have one solution and examples of equations that have infinitely many solutions. Let's take a look at one more type of equation. So we're going to solve this equation using the properties of equality. The first step is to distribute to remove the |
| 00:00:17 | parentheses. There are no parentheses on the left. So I'll just rewrite 5x minus 3. On the right, I need to distribute the 7, which will give me 7 times x plus 7 times 4, which is 28, minus 2x. Step two, we want to combine like terms on either side of the equal sign. There are no like terms on the left. |
| 00:00:37 | So just rewrite 5x minus 3 equals. On the right, we have like terms of 7x and negative 2x. Combining those gives us 5x plus 28. The third step is that we want to isolate the variable terms on one side of the equal sign and the constants on the other. So I'll start by using the subtraction property of equality to subtract 5x from both sides. |
| 00:01:00 | As you can see, our variable terms cancel out on both sides of the equal sign, leaving us with negative 3 equals 28. Obviously, negative 3 does not equal 28. They're two different numbers. So this is not a true statement. Anytime you end up with a false statement and no variables you can assume that you have no solution. So the answer to the question, what is the number of |
| 00:01:22 | solutions to this one-variable equation, is no solution. Take a look at step two. 5x minus 3 equals 5x plus 28. Note that our variable terms have the same coefficient, but our constants are different. Anytime you see that, there is no solution. |

**Section 8**

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| 00:00:01 | TEACHER: We're answering the question, how can you identify |
| 00:00:03 | the number of solutions of linear equations. You just learned how to solve equations with one solution, infinitely many solutions, and no solutions. Now you'll learn how to identify these equations and even write them. |

**Section 9**

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| 00:00:00 | TEACHER: Every linear equation can be simplified to an |
| 00:00:03 | equivalent equation of one of these three forms, x equals a, a equals a, or a equals b. And it's important to note that x is a variable, but a and b are constants. So what do these mean? Well, if we simplify our linear equation to the form x equals a, that means the equation is only true when the variable assumes the value of a. |
| 00:00:26 | And this means that there is only one solution. If we simplify the equation down to a equals a, that means the equation is true for any value of the variable, which means there are infinitely many solutions. And if we simplify down to the form of a equals b, that means there is no value of the variable that will make a equal to b, which means there are zero solutions. |

**Section 11**

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| 00:00:00 | TEACHER: Let's practice identifying |
| 00:00:01 | the number of solutions. So we're going to solve each equation and interpret the result. And our interpretation is going to be, how many solutions are there? So step one is to use the distributive property to eliminate the parentheses. So on the equation on the far left, I'll distribute the 2 on |
| 00:00:17 | the left-hand side and the 4 to the parentheses on the right. And that'll give me 2 times 2x is 4x, plus 2 times 10 is 20, equals 4 times x is 4x, minus 4 times 5 is 20, plus 6. So step two, I want to combine like terms on either side of the equals sign. There are no like terms on the left. 4x plus 20 equals-- |
| 00:00:44 | I have like terms of negative 20 and positive 6 on the right. And I can simplify that to negative 14. At this point, I can stop. Because notice that my variable terms have the same coefficient of 4. But my constants are not the same. The fact that my constants are not the same but my variable |
| 00:01:04 | coefficients are tells us that there are no solutions to this equation. So I would just write my interpretation as, no solution. All right, let's move on to the middle equation. Step one, distribute to eliminate the parentheses. So I'll distribute the 9 on the left. And there are no parentheses on the right. |
| 00:01:27 | This gives me 9 times x minus 9 times 2, which is 18, is equal to 9x minus 18. Well, again, I can stop at this step. Notice that I have my variable term on the left with a coefficient of 9, and my variable term on the right with a coefficient of 9. And I have the same constants on both sides, which are negative 18 and negative 18. |
| 00:01:50 | So the fact that my variable coefficients are the same and my constants are the same means any input value for x would satisfy the equation. And there are infinitely many solutions. OK, let's try the procedure on the third equation on the right. On the left, there are no parentheses. So I don't need to distribute. |
| 00:02:16 | I'll just rewrite 8x minus 4 minus 2x equals-- on the right, I'll distribute the 2. So 2 times 5x is 10x, and minus 2 times 2 is 4. And I'll go ahead and combine like terms on the left-hand side. So I have an 8x and a negative 2x. Combining those gives me 6x, minus 4 equals 10x minus 4. Step three, I need to get my variable term on one side of |
| 00:02:45 | the equation and my constants on the other side. So I'm going to start by using the subtraction property of equality and subtracting 6x from both sides. This leaves negative 4 equals 4x minus 4. And since my variable term is on the right side, I'll move my constants to the left using the addition property of equality-- add 4 to both sides. This gives me 0 equals 4x. |
| 00:03:14 | And the last thing I need to do is divide out the coefficient from both sides. So I'll use that division property of equality to divide 4 from both sides. 0 divided by 4 is 0, equals x, which is the same as x equals 0. So since I have my variable equal to a constant, there is exactly one solution. |
| 00:03:35 | And that is, x equals 0. |

**Section 13**

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| 00:00:00 | TEACHER: Let's practice writing an |
| 00:00:01 | equation with no solution. So we're going to follow these steps to create a linear equation with no solution. And step one says start with a statement in the form a equals b, where a and b, of course, are constant. So they can be any two numbers as long as they're different. So I'm going to start by saying 2 equals 6, even though I know that's not a true statement. |
| 00:00:21 | Remember, we're creating an equation with no solutions. So step two, add the same variable term to both sides. So choose a variable term. I'm going to choose 3x. I'm going to add 3x to the left-hand side. So 3x plus 2. And then, I'm going to add 3x to the right as well. 6 plus 3x. |
| 00:00:41 | Step three, add the same constant term to both sides. So pick a constant. Add it to both sides. I'll choose negative 10. That gives me negative 10 plus 3x plus 2. And now I need to add negative 10 to the right as well. Equals 6 plus 3x minus 10. So now I have an equation. |
| 00:01:01 | Let's verify that it has no solution. So I'll verify that by combining like terms on both sides. So on the left, I have like terms of negative 10 and positive 2, which are going to give me negative 8 plus 3x equals-- and on the right, I have like terms of 6 and negative 10, which when we combine those, gives me negative 4 plus 3x. |
| 00:01:24 | At this point, you can see that our variable term has the same coefficient of 3. And our constants are different, negative 8 and negative 4. So they should produce no solution. So let's go ahead and verify that this equation has no solution by using the properties of equality. So I'll use the subtraction property to subtract 3x from |
| 00:01:46 | both sides. And this shows that negative 8 equals negative 4, which of course is not true. We end up with a false statement, which means no solution. |